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Intraday Futures Volume and GARCH Effects in Heteroskedastic Mixture Model

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1. Introduction

THE Nikkei Index value is announced officially at every minute-by-minute during trading time, that is, from 9:01 through to 11:00 and from 13:01 through to 15:15. The index value is calculated by dividing the summation of the first value priced at every minute-by-minute of specified 225 stocks in the First Section of the Tokyo Stock Exchange with a divisor. The index value is announced officially after the next one minute of the summing. For example at 10:00 the announced index value is made from stock price which was first traded during 9:59. If one of 225 stocks has no trading during 9:59, then the stock price is selected the latest price before 9:59. However the prices of Nikkei futures and options are announced at every trading frequencies. Their markets had opened from 9:00 through to 11:15 and from 13:00 through to 15:15 before October 2, 1990 and then had changed at 9:00 to 11:00 and 13:10 to 15:15. Now they have changed at 9:00 to 11:00 and 13:10 to 15:00 since February 6, 1992. The lunch time break of the futures is 11:15 to 13:00. The officially announced time between the spot and the trading time of the derivatives is asynchronous so it is difficult to perfectly evidence the interactive behaviors between the spot and the futures, that is, the biggest lag between the spot pricing and the derivative pricing is one minute even when the futures is traded at every one minute.

Nakagawa (1992) reports that the open price of Nikkei Index is too late for information to reflect perfectly stock markets behavior because of asynchronous tradings between the spot and the futures, so the author considers evidently that the futures behavior reflects more Japanese security market than the spot behavior. Moreover Nakagawa (1995) argues, using Heteroskedastic Mixture Model (HMM)[1] that at least in nearby Nikkei futures markets before the crash of 1989, the daily shock of close-close log-relative prices influences significantly from the previous daily volumes and also GARCH effect has still a persistent power. In the overnight and daytime log-relative prices time series, the daily volumes influences significantly shocks of distant as well as nearby futures log-relative prices before the crash. Hence it is very interesting to investigate in detail with the time series of intraday futures log-relative prices. Chan, Chan, and Karolyi (1991) argue the interaction between S & P 500 Index and the futures with bivariate GARCH(1,3) with AR(1), using the data of every five minutes of their log-relative prices. However Locke and Sayers (1993) mention that GARCH model is misspecification and HMM with a no-lagged volume variable is more explainable the behavior of minute-by-minute S & P 500 Index futures pricing in April 1990 than GARCH model. This article is to evidence about the interrelationship between volume and GARCH effects in the intraday Nikkei futures markets.

The remainder of this paper is divided into four sections. The next section describes the data and basic statistics of nine futures. The following section is analyzed log-relative prices and volumes of every five minutes of Nikkei futures in HMM, compared with GARCH model. The final section provides a summary and conclusions.

2. Data and statistics

The price of Nikkei futures is announced at every trading frequencies. Respective futures prices consist of the time series of last three months before the mature month of each nearby futures. For example the price time series of the futures which mature in March 1989 is December 1st, 1988 to February 28, 1989. The analysis is used with the log-relative prices and volumes at every five minutes of respective futures. No trading day is excluded from the time series. The log-relative prices is so-called return. The futures return at time t is defined as follows.

$$r_t = 100 \cdot \ln(p_t / p_{t-1}), \tag{1}$$

where p_t denotes each futures price at time t. Table I shows the distributional statistics of returns of nine nearby futures which mature in March 1989 to March 1991.

The null hypotheses of zero sample means of all futures have no significance at five percent level. In the statistic result of daily returns in Nakagawa (1995) the twelfth and twentieth autocorrelations of most futures returns existed significantly in nearby and distant futures during both subperiods before and after the crash of 1989. However any intraday futures returns have no significant autocorrelations according to Ljung-Box Q-statistics,[2] except the futures which matures in March 1990. This evidence might results from that the interval of intraday prices which consist of returns is small, as compared with daily returns in Nakagawa (1995). According to the Ljung-Box Q-statistics of the autocorrelations of absolute and square returns, the unconditional distributions of all futures returns may be leptokurtic and also skew distribution because their each sample skewness and excess kurtosis[3] are not zeros. The sample skewness

Table I
Summary Statistics of Intraday of Nearby Nikkei Index Futures Log-Relative Prices

No trading day is excluded from the time series. Asymptotic t statistics are reported in parenthesis ( ) assuming the conditional normality. The ** (*) denotes that the coefficient is at 1 (5) percent level. $Q(n)$[1], $Q^*(n)$, and $Q^{**}(n)$ denote respectively the Ljung-Box Q-statistic for the nth-order serial correlation of log-relative prices, absolute log-relative prices, and square log-relative prices. The Ljung-Box Q-statistics subject to the asymptotically chi-square distribution.

Maturity	March '89	June	September	December	March '90	June	September	December	March '91
Number of observations	2123	3469	3693	3413	3241	3469	3693	3357	3056
Sample mean (%)	.0022024	.0012210	−0.00023459	.0018171	−0.0027485	−0.0014268	−0.0070789	−0.0045798	.0050430
	(1.73987)	(1.11062)	(−0.238181)	(1.77805)	(−1.11172)	(−0.384706)	(−1.94999)	(−0.804251)	(1.27234)
Sample variance (%²)	.0034051	.0041954	.0035844	.0035665	.019821	.047744	.048694	.10892	.048040
Sample skewness	.37936	−0.058905	−1.29983	−0.094162	−2.20428	−2.64241	−0.11265	4.57752	5.85333
Sample excess kurtosis	23.94381	31.62944	36.57255	141.62106	142.32830	75.09989	83.25218	201.37634	127.01936
Sample absolute mean	.033479	.038757	.036008	.032454	.062397	.11688	.092744	.11555	.097173
Sample square mean	.0034083	.0041957	.0035835	.0035687	.019823	.047732	.048731	.10891	.048049
$Q(12)$	19.9	6.94	7.74	26.2*	52.6**	30.0**	17.9	5.33	4.42
$Q(20)$	27.1	17.1	26.5	28.8	114**	38.4**	39.1**	10.7	9.40
$Q^*(3)$	39.2**	67.8**	125**	77.1**	30.9**	14.7**	1.84	22.9**	3.38
$Q^*(5)$	45.0**	84.2**	162**	96.1**	57.5**	38.2**	4.90	26.2**	4.04
$Q^{**}(3)$	3.36	3.79	3.62	1.39	.352	.092	.405	.194	.223
$Q^{**}(5)$	3.42	3.95	4.00	1.46	1.34	.572	.610	.292	.342

1) The Ljung-Box Q-statistic of the pth-order $Q(p)$ is as follows.

$$Q(p) = T(T+2)\sum_{i=1}^{p} \frac{1}{T-i}\rho_i^2,$$

where p denotes the order of the Ljung-Box Q-statistic, T denotes the number of observations, and ρ_i is the sample autocorrelation of lag i between residuals. See Ljung and Box(1978) in detail.

2) $\chi^2_{0.05}(3) = 7.814725$. $\chi^2_{0.05}(5) = 11.07048$. $\chi^2_{0.05}(12) = 21.02606$. $\chi^2_{0.05}(20) = 31.41042$. $\chi^2_{0.01}(3) = 11.34488$. $\chi^2_{0.01}(5) = 15.08632$. $\chi^2_{0.01}(12) = 26.21696$. $\chi^2_{0.01}(20) = 37.56627$. They denotes the chi-square statistics which the subscripts denote significant levels and the parentheses denote the degree of freedom.

and excess kurtosis correspond to the unconditional normal distribution as each value approachs zero. Thus it is available to evidence the behaviors of intraday returns with GARCH model which proposes the conditional normal distribution of returns. Moreover it is used for analyzing the relationship between GARCH and transaction volume effects in the HMM which involves a lagged volume variable as an exdogenous variable, in the conditional variance equation of disturbance term i.e. GARCH equation.

3. Volume and GARCH effects

The procedure which is identical to HMM in Nakagawa (1995) is denoted as follows.

$$r_t = c_0 + a_1 r_{t-1} + \varepsilon_t, \tag{2}$$

$$\varepsilon_t \mid \psi_{t-1} \sim N(0, h_t), \quad E[\varepsilon_t, \varepsilon_{t+1} \mid \psi_{t-1}] = 0, \quad \text{for} \quad i = 1, 2, \cdots,$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \phi_1 V_{t-1}, \tag{3}$$

where

$$\psi_t = \{V_t, V_{t-1}, \cdots, \varepsilon_t, \varepsilon_{t-1}, \cdots\}. \tag{4}$$

r_t denotes a futures return at time t. ε_t denotes the disturbance term of intraday returns at time t and implies the intraday unpredictable return which is interpreted as a collective measure of news from time t–1 to time t. V_t denotes the intraday trading volumes from time t–1 to time t. Let ψ_t be the past bivariate information set which consists of the past intraday unpredictable returns and trading volumes up to time t. h_t is the conditional (i.e. time-varying) variance of the disturbance term of intraday returns and calls as the shock of intraday returns at time t, and $E[\cdot \mid \psi_{t-1}]$ denotes the conditional expectation, given the bivariate information set at time t–1. $N(0, h_t)$ denotes the normal distribution that the mean is zero and the time-varying variance is h_t. c_0 and α_0 denote constant terms. a_1, α_1, β_1 and ϕ_1 denote coefficient terms. The parameter space is constrained to be nonnegative and is estimated jointly using numerical technique to maximize the log-likelihood function. The estimating method is Berndt, Hall, Hall, and Hausman (BHHH) (1974).

Table II shows the estimates of GARCH model and HMM. The autoregressive order of the conditional expectation models in Table II was determined by the statistics of Lagrange Multiplier, Likelihood Ratio, and Wald tests. These statistics have a relationship that Lagrange Multiplier statistic is the biggest of all and Likelihood Ratio statistic is bigger than Wald statistic.[4] In other words the hypotheses which each conditional expectation model was adequate with only constant term or AR(1) were judged by three tests that subjected to the asymptotic chi-square distribution.[5] Also a_1 was not significant by three tests if and only if the a_1 was not significant by t-statistics in the conditional expectation equation. Finally the hypotheses which each conditional variance equation of disturbance term was GARCH model or HMM were also judged by three tests.

From the estimates of GARCH models in Table II, futures which mature before the crash of December 1989, that is, in December 1989 to February 1990 have significant GARCH effects which are measured by $\alpha_1 + \beta_1$. This fact implies that the shocks of intraday returns in five futures may change at every five minutes before the crash. After the crash, however the shocks of returns may be constant in GARCH models because the shocks are measured by α_0 s which have no significant relationships between the previous shocks. This evident result is inconsistent with the significant GARCH effects of nearby daily (i.e. close-close, and overnight and daytime) returns in Nakagawa (1995). According to the statistics of Lagrange Multiplier, Likelihood Ratio, and Wald tests, HMMs might be more explainable the behaviors of all intraday futures returns than GARCH models. Moreover it is very interesting that volume effects are significant at one percent level in periods after as well as before the crash, despite no GARCH effect is significant in HMMs after the crash. Similar to the results of Najand and Yung (1991) and Nakagawa (1995), the GARCH effects still have significantly and persistently in futures markets which mature before the crash of 1989. The GARCH effects before the crash are persistent, however in HMMs which futures mature in March and September 1989 the GARCH effects are weaker than ones in GARCH models, similar to the results of Lamoureuy and Lastrapes (1990). It is also worth noting that futures which mature in June 1989 and March 1990 have no GARCH effects in GARCH models, despite in HMMs they have significant GARCH effects. This evident result of the volume effects before the crash is consistent significantly with the result of the nearby daily returns in Nakagawa (1995).

Hence it is worth investigating in detail the behaviors of the futures returns before the crash. So it is available to describe the distribution of every five minute returns and trading volumes before analyzing the behaviors of returns. Wood, McInish, and Ord (1985) describe the distribution of minute-by-minute return which is an equally weighted index of common stocks listed on the NYSE in September 1971 to February 1972 and 1982. They report that abnormal high returns and standard deviations of returns are found at the beginning and the end of the trading day. Adati and Pfleiderer (1988) show only the average trading volume data of the beginning, the middle, and the end of the trading day at every two hours. They call the histogram of the average volume as the U-shaped pattern of shared traded volumes in US markets because the average trading volumes of the beginning and the end of the trading day are bigger than one of the middle of the trading day. As a result of investigating the average returns and trading volumes of every five minutes of all Nikkei futures with figures, the behaviors of futures which mature in March and June 1989 are shown in Figure because the volume behaviors of the other futures are the almost same pattern as the behaviors in the figure. In Figure, the *black block* and the *gray block* denote respectively the average trading volumes of every five minutes which futures matures in March and June 1989. The *solid line* and the *dotted line* denote respectively the average returns of every five minutes. The trading volumes of the beginning and the end of both *Zem-Ba* and *Go-Ba* are bigger than volumes of the other trading hours in Nikkei futures markets.[6] There is lunch time

Table II
Maximum Likelihood Estimates of GARCH(1,1) and Heteroskedastic Mixture Model of Intraday Nikkei Index Futures Log-Relative Prices

No trading day is excluded from the time series. Asymptotic t statistics are reported in parenthesis ( ) assuming the conditional normality. The ** (*) denotes that the coefficient is at 1 (5) percent level. First the hypotheses which the conditional expectation model was adequate only constant term or AR(1) were judged by Lagrange Multiplier, Likelihood Ratio, and Wald statistics. Second the hypotheses which each h_t is GARCH model or HMM are judged by the three tests. Their three statistics subject respectively to the asymptotic chi-square distribution.

$$r_t = c_0 + a_1 r_{t-1} + \varepsilon_t, \quad \varepsilon_t \mid \psi_{t-1} \sim N(0, h_t), \quad E[\varepsilon_t \varepsilon_{t+j} \mid \psi_{t-1}] = 0, \quad (j = 1, 2, \cdots),$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \phi_1 V_{t-1}.$$

c_0	a_1	α_0	α_1	β_1	ϕ_1	$\alpha_1+\beta_1$	Log-likelihood
Maturity : March '89 (Number of observations = 2123) Lagrange Multiplier = 537.27**, Likelihood Ratio = 619.46**, Wald = 719.31**.							
$.209459\times10^{-2}$		$.203708\times10^{-2}$**	.067333**	.344762**		.412095	3030.65
(1.65842)		(6.76108)	(4.72480)	(3.62549)			
$.390771\times10^{-3}$		$.102520\times10^{-2}$**	.211398**	.149096**	$.596243\times10^{-5}$**	.360494	3340.38
(.422418)		(26.3062)	(11.9928)	(8.82058)	(29.4846)		
Maturity : June '89 (Number of observations = 3469) Lagrange Multiplier = 1174.78**, Likelihood Ratio = 1434.34**, Wald = 1776.34**.							
$.115482\times10^{-2}$		$.417904\times10^{-2}$**	.010490	0		.010490	4572.79
(1.03216)		(168.954)	(1.58804)	(0)			
$.108201\times10^{-2}$		$.612872\times10^{-3}$**	.063883**	.237630**	$.978606\times10^{-5}$**	.301513	5289.96
(1.39436)		(24.3494)	(7.05551)	(23.8344)	(26.1889)		
Maturity : September '89 (Number of observations = 3693) Lagrange Multiplier = 1134.95**, Likelihood Ratio = 1356.04**, Wald = 1638.49**.							
-0.444090×10^{-3}		$.593297\times10^{-3}$**	.081133**	.767062**		.848195	5222.39
(−0.464278)		(10.1593)	(11.5386)	(36.0944)			
-0.134596×10^{-2}		$.460927\times10^{-3}$**	.055379**	.332405**	$.542217\times10^{-5}$**	.387784	5900.41
(−1.75433)		(39.5523)	(5.64135)	(46.4918)	(23.7971)		
Maturity : December '89 (Number of observations = 3413) Lagrange Multiplier = 1792.76**, Likelihood Ratio = 2542.74**, Wald = 3776.4**.							
$.174307\times10^{-2}$		$.354222\times10^{-2}$**	.029439**	0		.029439	4785.18
(1.67348)		(349.844)	(2.91534)	(0)			
$.851264\times10^{-3}$	−0.049513**	$.320708\times10^{-3}$**	.12786**	$.211791\times10^{-2}$**	$.661343\times10^{-5}$**	.12980391	6056.55
(1.39714)	(−2.74220)	(16.4177)	(10.5024)	(.338685)	(37.6110)		

Maturity : March '90 (Number of observations = 3241) Lagrange Multiplier = 857.52**, Likelihood Ratio = 996.02**, Wald = 1166.03**.							
-0.259378×10^{-2}	.042549*	.019816**	$.313005\times10^{-2}$	0		$.313005\times10^{-2}$	1755.48
(−1.02694)	(2.06514)	(336.213)	(.655642)	(0)			
-0.273768×10^{-2}	.062495**	.010176**	.016947**	0	$.122758\times10^{-5}$**	.016947	2253.49
(−0.952451)	(2.83407)	(373.896)	(2.42876)	(0)	(77.4888)		
Maturity : June '90 (Number of observations = 3469) Lagrange Multiplier = 168.310**, Likelihood Ratio = 172.530**, Wald = 176.890**.							
-0.13596×10^{-2}	.052196**	.047654**	0	0		0	357.521
(−0.347691)	(2.91704)	(246.082)	(0)	(0)			
-0.159528×10^{-2}	.053103**	.040971**	0	0	$.808890\times10^{-5}$**	0	443.786
(−0.405171)	(3.08386)	(268.301)	(0)	(0)	(15.9879)		
Maturity : September '90 (Number of observations = 3693) Lagrange Multiplier = 210.280**, Likelihood Ratio = 216.504**, Wald = 222.976**.							
-0.677312×10^{-2}	.044593*	.048628**	0	0		0	343.348
(−1.86548)	(1.97074)	(280.195)	(0)	(0)			
-0.549171×10^{-2}	.043011*	.041537**	0	0	$.640537\times10^{-5}$**	0	451.600
(−1.54703)	(1.98840)	(312.381)	(0)	(0)	(22.0252)		
Maturity : December '90 (Number of observations = 3357) Lagrange Multiplier = 39.822**, Likelihood Ratio = 40.060**, Wald = 40.300**.							
-0.459657×10^{-2}		.108988**	0	0		0	−1042.63
(−0.763896)		(391.153)	(0)	(0)			
-0.332803×10^{-2}		.105942**	0	0	$.536225\times10^{-5}$**	0	−1022.60
(−0.535246)		(395.200)	(0)	(0)	(9.03545)		
Maturity : March '91 (Number of observations = 3056) Lagrange Multiplier = 586.070**, Likelihood Ratio = 650.674**, Wald = 725.133**.							
$.504851\times10^{-2}$		.048071**	0	0		0	301.221
(1.09065)		(268.960)	(0)	(0)			
$.539729\times10^{-2}$		.033244**	0	0	$.697063\times10^{-5}$**	0	626.558
(1.03886)		(281.445)	(0)	(0)	(32.3068)		

1) $t_{0.05}(2118) = 1.961084$. $t_{0.05}(3689) = 1.960607$. $t_{0.01}(2118) = 2.578163$. $t_{0.01}(3689) = 2.577162$. They denote the t-statistics which the subscripts denote significant levels and the parentheses denote the degree of freedom.
2) $\chi^2_{0.05}(1) = 3.841455$. $\chi^2_{0.05}(2) = 6.634891$. $\chi^2_{0.01}(1) = 5.991476$. $\chi^2_{0.01}(2) = 9.210351$. They denote the chi-square statistics which the subscripts denote significant levels and the parentheses denote the degree of freedom.

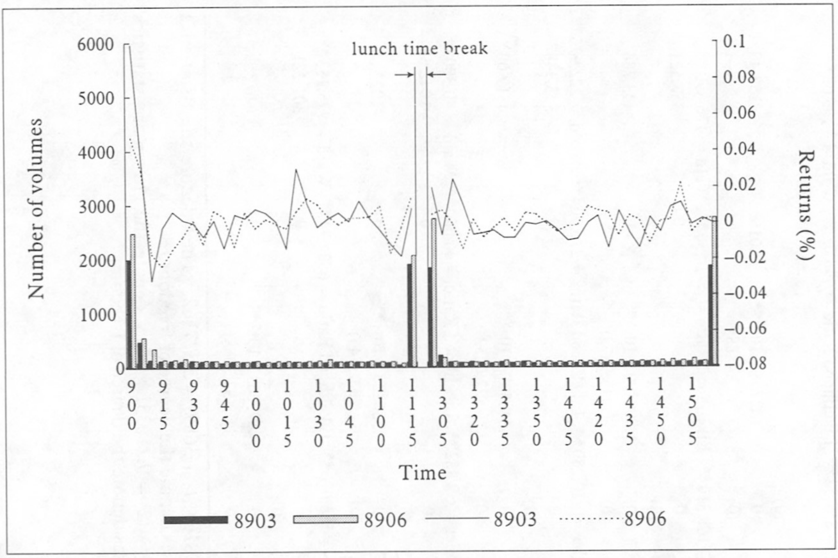

Figure. Sample average returns and trading volumes during every five minutes

No trading day is excluded from Figure. The *black block* and the *gray block* denote respectively the average trading volumes of every five minutes which futures matures in March and June 1989. The *solid line* and the *dotted line* denote respectively the average returns of every five minutes which futures matures in March and June 1989. The lunch time break is from 11:15 through to 13:00. The sample days which a futures matures in March 1989 are 39 days, the average volumes are 229.03, the sample standard deviation of trading volumes is 470.66, the average return is 0.022024 percent, and the sample standard deviation of returns is 0.017256. The sample days which a futures matures in June 1989 are 62 days, the average and the standard deviation of volumes are 292.76 and 629.95 respectively, the average and the standard deviation of returns are respectively 0.012210 percent and 0.011203.

break in most Japan's security markets, so the histograms of the futures volumes are called as the W-shaped patterns of shared traded volumes and might be a characteristic of Japanese stock markets.

Chan, Chan, and Karolyi (1991) argue the interaction between S & P 500 Index and the futures with bivariate AR(1)-GARCH(1,3) with two dummy variables which correspond to overnight and the first five minutes after opening stock market. In this paper, a modified model is developed by new HMM with four dummy variables because of the W-shaped pattern of the volumes. The HMM is the following model which includes four dummy variables.[7]

$$r_t = c_0 + \sum_{i=1}^{4} c_1 D_{it} + \varepsilon_t, \tag{5}$$

$$\varepsilon_t \mid \psi_{t-1} \sim N(0, h_t), \quad E[\varepsilon_t \varepsilon_{t+i} \mid \psi_{t-1}] = 0, \quad \text{for} \quad i = 1, 2, \cdots,$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \left(\left(1 - \sum_{i=1}^{4} D_{it} \right) \phi_0 + \sum_{i=1}^{4} \phi_i D_{it} \right) V_{t-1}, \tag{6}$$

$$D_{1t} = \begin{cases} 1 & \textit{if} \quad t = 9{:}00, \\ 0 & \textit{otherwise}, \end{cases} \quad D_{2t} = \begin{cases} 1 & \textit{if} \quad t = 9{:}05, \\ 0 & \textit{otherwise}, \end{cases}$$
$$D_{3t} = \begin{cases} 1 & \textit{if} \quad t = 13{:}00, \\ 0 & \textit{otherwise}, \end{cases} \quad D_{4t} = \begin{cases} 1 & \textit{if} \quad t = 13{:}05, \\ 0 & \textit{otherwise}. \end{cases} \tag{7}$$

c_i for $i = 0, 1, \cdots, 4$ and α_0 denote constant terms. a_1, β_1, and ϕ_i for $i = 0, 1, \cdots, 4$ denote coefficient terms. D_{it} s for $i = 1, \cdots, 4$ denote dummy variables. D_{1t} denotes the dummy variable which takes a value of 1 on overnight returns and a value of 0 on otherwise, and D_{3t} denotes the dummy variable which takes a value of 1 on lunch time break returns and a value of 0 on otherwise. D_{2t} denotes the dummy variable which takes a value of 1 on the first five minutes returns after opening futures market and a value of 0 on otherwise. D_{4t} denotes the dummy variable which takes a value of 1 on the first five minutes returns after lunch time break and a value of 0 on otherwise. The parameter space of the conditional variance of disturbance term equation is constrained to be nonnegative and is estimated jointly using numerical technique to maximize the log-likelihood function. The estimating method is BHHH. The hypotheses which each HMM is more explainable than each GARCH model in Table II were judged by the above three tests in Table III.

Table III
Maximum Likelihood Estimates of Heteroskedastic Mixture Model with Four Dummy Variables of Intraday Nikkei Index Futures Log-Relative Prices

No trading day is excluded from the time series. D_{1t} denotes the dummy variable which takes a value of 1 on overnight returns and a value of 0 on otherwise, and D_{3t} denotes the dummy variable which takes a value of 1 on lunch time break returns and a value of 0 on otherwise. D_{2t} denotes the dummy variable which takes a value of 1 on the first five minutes returns after opening markets and a value of 0 on otherwise. D_{4t} denotes the dummy variable which takes a value of 1 on the first five minutes returns after lunch time break and a value of 0 on otherwise. Asymptotic t statistics are reported in parenthesis ( ) assuming the conditional normality. The ** (*) denotes that the coefficient is at the 1 (5) percent level. The hypotheses which each HMM in Table III is more explainable than each GARCH model in Table II are judged by Lagrange Multiplier, Likelihood Ratio, and Wald statistics. Their three statistics subject respectively to the asymptotic chi-square distribution.

$$r_t = c_0 + \sum_{i=1}^{4} c_i D_{it} + \varepsilon_t, \quad \varepsilon_t \mid \psi_{t-1} \sim N(0, h_t), \quad E[\varepsilon_t \varepsilon_{t+j} \mid \psi_{t-1}] = 0, \quad (j = 1, 2, \cdots),$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \left(\left(1 - \sum_{i=1}^{4} D_{it} \right) \phi_0 + \sum_{i=1}^{4} \phi_i D_{it} \right) V_{t-1},$$

$$D_{1t} = \begin{cases} 1 & if \quad t = 9{:}00, \\ 0 & otherwise, \end{cases} \quad D_{2t} = \begin{cases} 1 & if \quad t = 9{:}05, \\ 0 & otherwise, \end{cases} \quad D_{3t} = \begin{cases} 1 & if \quad t = 13{:}00, \\ 0 & otherwise, \end{cases} \quad D_{4t} = \begin{cases} 1 & if \quad t = 13{:}05, \\ 0 & otherwise. \end{cases}$$

Maturity	March '89	June	September	December
c_0	-0.737422×10^{-3}	$.110833\times10^{-3}$	-0.961859×10^{-3}	$.778782\times10^{-3}$
	(−0.812739)	(.140166)	(−1.43547)	(1.23710)
c_1	.131755**	.106753**	.031529	.041852**
	(9.60032)	(3.88936)	(1.28366)	(2.64089)
c_2	−0.197245**	-0.599198×10^{-2}	.016257	$.551516\times10^{-2}$
	(−16.9241)	(−0.437086)	(.490907)	(.276306)
c_3	.015858	$.591350\times10^{-2}$	-0.897591×10^{-2}**	$.803099\times10^{-2}$
	(.928544)	(.849475)	(−3.02848)	(1.18231)
c_4	$.720542\times10^{-2}$	$.896976\times10^{-2}$	-0.897591×10^{-2}	−0.015500**
	(.397776)	(.913079)	(−1.30127)	(−3.05888)

α_0	$.887658\times10^{-3}$**	$.658300\times10^{-3}$**	$.242845\times10^{-3}$**	$.329327\times10^{-3}$**
	(21.3273)	(19.1539)	(9.64191)	(11.7409)
α_1	.183293**	.035914**	.096911**	.111460**
	(8.60140)	(3.06502)	(6.61413)	(6.17048)
β_1	.092173**	.184721**	.316234**	$.224306\times10^{-3}$
	(3.86508)	(3.06502)	(11.3404)	(0.12715)
ϕ_0	$.700495\times10^{-5}$**	$.116980\times10^{-4}$**	$.683626\times10^{-5}$**	$.711760\times10^{-5}$**
	(20.0920)	(27.2710)	(20.4184)	(31.5159)
ϕ_1	$.200171\times10^{-5}$*	$.799238\times10^{-5}$**	$.169284\times10^{-4}$**	$.172717\times10^{-5}$**
	(2.54153)	(5.82751)	(5.68350)	(5.18710)
ϕ_2	$.943102\times10^{-5}$**	$.149484\times10^{-5}$**	$.130078\times10^{-6}$	$.154512\times10^{-5}$**
	(3.46019)	(3.17360)	(.657479)	(6.09063)
ϕ_3	$.333168\times10^{-6}$**	0	$.204519\times10^{-6}$**	$.164639\times10^{-6}$**
	(2.95299)	(0)	(3.87535)	(3.38950)
ϕ_4	0	$.781679\times10^{-7}$	$.628579\times10^{-8}$	$.106331\times10^{-7}$
	(0)	(1.45150)	(.184698)	(.573990)
Log-likelihood	3581.18	5632.97	6331.20	6298.56
Lagrange Multiplier	430.88**	622.45**	768.46**	451.27
Likelihood Ratio	481.60**	686.02**	861.58**	484.02
Wald	979.88**	1555.23**	1715.56**	1446.88

1) $t_{0.05}(2110) = 1.961089$. $t_{0.05}(3400) = 1.960661$. $t_{0.05}(3456) = 1.960652$. $t_{0.05}(3680) = 1.960607$. $t_{0.01}(2110) = 2.578163$. $t_{0.01}(3400) = 2.577272$. $t_{0.01}(3456) = 2.577253$. $t_{0.01}(3680) = 2.577162$. They denote t-statistics which the subscripts denote significant levels and the parentheses denote the degree of freedom.

2) $\chi^2_{0.01}(8) = 20.09016$. It denotes the chi-square statistic which the subscript denotes significant level and the parenthesis denotes the degree of freedom.

In Table III, all futures persist significantly GARCH effects, and also have significantly overnight volume effects which are evaluated by ϕ_1. However all futures markets have no significant volume effects of the first five minutes after lunch time break, in spite of the volumes during the time is bigger than the average trading volume. This implies that GARCH models may be better than HMMs in the behaviors of all futures returns from 13:00 to 13:05. In the other volume effects, futures which mature in March and December have significant ϕ_2 and ϕ_3 which imply volume effects during the first five minutes after opening markets and lunch time break. On the other hand, either ϕ_2 or ϕ_3 which implies volume effect of either first five minutes after opening the markets or lunch time break is significant in futures which mature in June and September.

4. Conclusion

Nakagawa (1995) argued, using Heteroskedastic Mixture Model (HMM), that the daily shock of close-close returns influenced significantly from the previous daily volume and also had still a persistent GARCH effect, in nearby Nikkei futures markets before the crash of December 1989. In overnight and daytime returns time series, the daily volume of distant as well as nearby futures influenced significantly shocks of respective futures returns before the crash. Hence it is very interesting to investigate in detail with intraday returns of nearby futures.

The first analysis is used to analyze volume and GARCH effects in HMM with the returns of every five minutes of nine nearby futures of March 1989 to March 1990. No trading day was excluded from the time series. The hypotheses which each conditional expectation equation in HMM was adequate with only constant term or AR(1) were judged by the statistics of Lagrange Multiplier, Likelihood Ratio, and Wald tests that subjected to the asymptotic chi-square distribution. Finally the hypotheses which each conditional variance equation of disturbance term was GARCH model or HMM were also judged by the three tests. As a result of in GARCH models of five futures before the crash of December 1989, the shocks of intraday returns may change by every five minutes. However HMMs might be more explainable the behaviors of all futures returns than GARCH models. Moreover it is interesting that volume effects are significant at one percent level in periods after as well as before the crash, despite no GARCH effect is significant in both HMMs and GARCH models after the crash. In GARCH models this fact is inconsistent with the significant GARCH effects of daily nearby returns in Nakagawa (1995). However the evident result of the volume effects before the crash is consistent significantly with the result of the nearby daily returns in Nakagawa (1995).

Hence it is worth evidencing in detail the behaviors of the futures returns before the crash, using HMM with four dummy variables. All futures persist significantly GARCH effects, and also significantly overnight volume effects. It is interesting that all futures markets have no significant volume effects during the first five minutes after lunch time break, in spite of the volumes during the time are bigger than the average trading volumes. This implies that GARCH models may be better than HMMs in the behaviors of futures returns from 13:00 to 13:05. Futures which mature in March and December 1989 have significant volume effects during the first five minutes of opening markets and lunch time break. Also volume effect of either the first five minutes after opening markets or lunch time break is significant in futures which mature in June and September. In general before the crash of 1989 the behavior of intraday futures returns is similar to one of daily nearby futures returns in Nakagawa (1995), since both volume and GARCH effects are persistent and significant in HMM. However after the crash there are no significant GARCH effects of intraday returns and daily nearby returns.

Notes

[1] The Heteroskedastic Mixture Model is mentioned by Najand and Yung (1991) and Nakagawa (1995) in detail.

[2] The Ljung-Box Q-statistic of the pth-order $Q(p)$ is as follows.

$$Q(p) = T(T+2)\sum_{i=1}^{p} \frac{1}{T-i}\rho_i^2,$$

where p denotes the order of the Ljung-Box Q-statistic, T denotes the number of observations, and ρ_i is the sample autocorrelation of lag i between residuals. See Ljung and Box (1978) in detail.

[3] The sample skewness and the sample excess kurtosis are the followings respectively.

$$\frac{T}{(T-1)(T-2)}\sum_{t=1}^{T}\left(\frac{r_t-\mu_r}{s_r}\right)^3,$$

$$\frac{T(T+1)}{(T-1)(T-2)(T-3)}\sum_{t=1}^{T}\left(\frac{r_t-\mu_r}{s_r}\right)^4 - \frac{3(T-1)^2}{(T-2)(T-3)},$$

where T denotes the sample size, μ_r denotes the sample average of returns, and s_r denotes the sample standard deviation of returns. Usually the sample kurtosis denotes the value which deletes the second term in the second equation.

[4] See Berndt and Savin (1977) and Engle (1984) in detail.

[5] Therefore the Lagrange Multiplier test is the strongest test against the null hypothesis.

[6] The *Zem-Ba* is called as the opened market from 9:00 through to 11:15 before lunch time break and the *Go-Ba* is called as the opened market from 13:00 through to 15:15 after lunch time break.

[7] The following model was first estimated.

$$r_t = c_0 + \sum_{i=1}^{4} c_i D_{it} + \varepsilon_t,$$

$$\varepsilon_t \mid \psi_{t-1} \sim N(0, h_t), \quad E[\varepsilon_t \varepsilon_{t+i} \mid \psi_{t-1}] = 0, \quad \text{for} \quad i = 1, 2, \cdots$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \left(\phi_0 + \sum_{i=1}^{4}\phi_i D_{it}\right) V_{t-1}$$

$$D_{1t} = \begin{cases} 1 & \textit{if} \quad t = 9{:}00, \\ 0 & \textit{otherwise}, \end{cases} \quad D_{2t} = \begin{cases} 1 & \textit{if} \quad t = 9{:}05, \\ 0 & \textit{otherwise}, \end{cases}$$

$$D_{3t} = \begin{cases} 1 & \textit{if} \quad t = 13{:}00, \\ 0 & \textit{otherwise}, \end{cases} \quad D_{4t} = \begin{cases} 1 & \textit{if} \quad t = 13{:}05, \\ 0 & \textit{otherwise}, \end{cases}$$

However some coefficient estimates of dummy variables in the conditional variance equation of error term were zeros. It is the reason that the parameter space is constrained to be nonnegative, so was changed eq. (5) and eq. (6).

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